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A DISSERTATION ON THE GEOMETRICAL ANALYSIS OF THE ANTIENTS.

WITH A

Collection of THEOREMS and PROBLEMS, WITHOUT SOLUTIONS,

FOR THE

EXERCISE of YOUNG STUDENTS.

FUNGAR VICE COTIS.

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BRITISH MUSEUM

A DISSERTATION ON THE GEOMETRICAL ANALYSIS OF THE ANTIENTS.

IT is univerſally allowed that Mathematical Studies are attended with much pleaſure and delight, and that they are in a particular manner captivating and engaging to thoſe whoſe genius and bent of mind lie that way; and it is equally certain that they are profitable as well as pleaſant. From the proper cultivation of them many emoluments accrue to ſociety, and many improvements are made in civil life. But ſetting aſide theſe public advantages, for which the world muſt be indebted to men of ſuperior ability and exalted genius, qualified by Nature to extend the bounds of Science, and to derive

practical improvements from their abſtract reaſonings; I would now only recommend theſe ſtudies, as uſeful to men of all ranks, profeſſions and abilities, in their private capacities. They help greatly to fix the attention, to enlarge the underſtanding, to methodiſe our ideas, and to improve our reaſoning faculties, when we apply them to any other ſcience whatſoever. Great regard was had to theſe ſciences by the antients in their education of children; and they ought certainly to have a principal ſhare in the education of gentlemen and ſcholars, if we would have them taught to think juſtly or to reaſon well. Of all the branches of Mathematical Science none conduces more to the great ends here mentioned than Geometry. The ſimplicity of it's firſt principles, the clearneſs and evidence of it's demonſtrations, the admirable concatenation of it's parts, and regular connexion of the propoſitions with one another, tend greatly to eſtabliſh a habit of cloſe thinking, and a methodical and juſt argumentation, when we apply ourſelves to any other ſubject whatſoever, phyſical or moral, œconomical or civil.

But

But I know not how it happens, this noble ſcience ſeems of late to have met with leſs regard than it's dignity and uſefulneſs demand. Our ſchools and univerſities*, our philoſophical ſocieties, and philomaths of all degrees, ſeem to have been very aſſiduous of late in paying their devoirs to the younger ſiſter *Algebra*; and at the ſame time to have overlooked, and in a great meaſure neglected, thoſe native charms, that amiable ſimplicity, and thoſe more attractive excellencies, that peculiarly belong to the elder. I would not be thought any ways to diſparage the *algebraical* art, or to derogate from it's merits; they are doubtleſs very great. All the branches of the Mathematics are much indebted thereto; and even Geometry itſelf has within this laſt century been ſurpriſingly extended and improved thereby. But, notwithſtanding this, I muſt affirm that, for thoſe who apply themſelves to the Mathematics only for the ends above-mentioned, namely, to inure themſelves to a method of cloſe thinking and

* *The reader is referred to ſome late reſolutions taken in the univerſity of Cambridge.*

juſt reaſoning, *Geometry* is the proper field: and that thoſe who but moderately exerciſe themſelves therein will ſooner attain their purpoſe, than others that may go far greater lengths in the intricacies of *Algebra*, and the labyrinth of *Fluxions*.

It is but too common a practice with young ſtudents, after having gone through the Elements of *Euclid* in a curſory manner, (or perhaps having ſubſtituted in their place ſome other *ſuperficial* and *leſs geometrical* elements,) from henceforth to bid adieu to all demonſtrations ſtrictly geometrical, and to employ themſelves in the conſideration of algebraical equations, infinite ſeries, the algorithm of fluxions, the properties of curve lines, and other parts of the *higher* Geometry, when perhaps they are but very ſuperficially acquainted with the firſt elements of the *plane*. If they read a ſyſtem of conics, they will be ſure to make choice of one, where the demonſtrations are algebraical; and when they apply themſelves to the ſolution of geometrical problems, here again they will have recourſe to their

their Algebra, not at all apprized perhaps of any other analyſis; being utterly unacquainted with the method of reſolution and compoſition ſo carefully cultivated by the antient geometricians. Such ſtudents may become dextrous calculators, but ſuch a method of proceeding tends very little towards bettering the judgment, or improving the invention.

The ſtudy of Geometry I ſay is the moſt proper for young men to purſue, in order to acquire a vigorous conſtitution of mind, and is as conducive thereto as exerciſe is towards procuring health and ſtrength to the body. *Logical* precepts are uſeful, and indeed neceſſary for thoſe that are engaged in public diſputations, or controverſial writings, in order to put to ſilence an obſtinate adverſary. But 'in the ſearch of 'truth, an imitation of the method of *Geometers* 'will carry a man further than all the *dialectical* 'rules. Their *Analyſis* is the proper model we 'ought to form ourſelves upon, and imitate in 'the regular diſpoſition, and gradual progreſs 'of our enquiries *.'

* *Eſſay on the uſefulneſs of Mathematical Learning. Oxf.* 1701.

We

We are told by Dr. *Pemberton**, 'that Sir 'Iſaac Newton uſed to cenſure himſelf for not 'following the Antients more cloſely than he 'did; and ſpoke with regret of his miſtake, 'at the beginning of his mathematical ſtudies, 'in applying himſelf to the works of *Deſ-* 'cartes, and other algebraical writers, before 'he had conſidered the Elements of *Euclid* with 'that attention ſo excellent a writer deſerves. 'That he highly approved the laudable at- 'tempt of *Hugo de Omerique* to reſtore the 'antient *Analyſis*.' Now what the great Sir *Iſaac Newton* ſo highly approved, it is the intention of this publication more particularly to ſpecify and recommend. Little has yet been done toward the attainment of this laudable purpoſe of reſtoring the antient Analyſis. The writer juſt mentioned is very little known in England. The Author of this ſmall Tract is willing to contribute his mite, and very deſirous to revive a proper taſte for pure Geometry. He has annexed a collection of *Theorems* and likewiſe a few *Problems*, to be ſolved by the

* *In the preface to his View, &c.*

Geometrical Analyſis: he has been more ſparing in the latter, becauſe plenty of them are continually propoſed in periodical publications. It is not pretended that they are new ones; but they are ſuch as rarely occur to them for whoſe uſe they are principally intended. Not above four or five of them, I believe, have ever appeared in Engliſh before; and they are all taken from authors which ſeldom fall into the hands of young men. They will ſerve therefore as proper exerciſes for young ſtudents to try their ſtrength upon.

But before they ſet themſelves to this work, I would recommend a very careful and reiterated peruſal of the Elements, and after that as diligent an application to that valuable remains of antiquity, the book of *Euclid's Data*, both which they will find moſt compleat in Dr. *Robert Simſon's* edition. When they have made themſelves perfect maſters of theſe, they may then betake themſelves to the ſolution of Geometrical Propoſitions by a Geometrical Analyſis; either that of the Antients derived from the Data: or, if this ſhould be thought too tedious

tedious and troubleſome, they may abate ſomewhat of it's rigour, and ſtill make uſe of a ſimilar method: but I would have them by no means content themſelves with algebraical reſolutions, even though they ſhould be able to derive conſtructions from thence, and alſo to demonſtrate ſynthetically the truth of the ſame. How they may proceed with ſucceſs I ſhall endeavour briefly to explain.

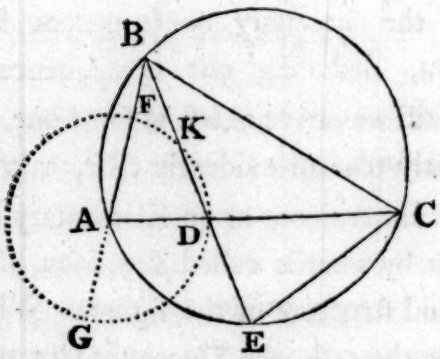
Reſolution then or *Analyſis* is the method of proceeding from the thing *ſought* as *taken* for granted through it's conſequences to ſomething that is *really* granted; and *Compoſition* or *Syntheſis* is a reverſe method, wherein we lay that down firſt which was the laſt ſtep of the *Analyſis*, and tracing the ſteps of the *Analyſis* back, making that antecedent here which was conſequent there, till we arrive at the thing *ſought*, which was put as granted in the firſt ſtep of the Analyſis.

When we are to apply this method of *Reſolution* to *Theorems*, we muſt firſt lay what is therein affirmed down as true, and then conſider

conſider the neceſſary conſequences flowing therefrom, deducing one conſequence from another, till we arrive at laſt at ſome one, which is evidently true or evidently falſe, as may appear by an Axiom, or an Elementary Propoſition, or by what is called *Expoſition*, i. e. the nature and ſtructure of the figure. When the former is the caſe, the *Theorem* is true and may be demonſtrated by the method of *Compoſition*, but when the latter is the caſe, it is falſe, for all truths are conſiſtent with each other. An example will clear this more than many words.

THEOREM.

The ſquare of a line biſecting the vertical angle of any triangle, together with the rectangle under the ſegments of the baſe made thereby, is equal to the rectangle under the ſides containing that angle.



ANALYSIS.

Suppoſe this to be true, viz. that $BD^2 + ADC = AB \times BC$, and let a circle be circumſcribed about the triangle, and BD produced to meet it in E, and EC joined. Now ADC = BDE by Euc. III. 35. Therefore $BD^2 + ADC = BD^2 + BDE = EBD$ by II. 3. Therefore alſo $AB \times BC = EBD$. Now this we ſhall find to be true by the Elements, hence the theorem is alſo true. For the triangles ABD and EBC are ſimilar, having the angles at A and E equal as ſtanding on the ſame circumference and the angles at B in each equal by *Expoſition*, therefore by VI. 4. AB : BD :: BE : BC and by VI. 16. $AB \times BC = EBD$.

SYNTHESIS.

SYNTHESIS.

$AB \times BC = EBD$ (as proved in the Analyſis)
$= BD^2 + BDE$ by II. 3. $= BD^2 + ADC$ by III. 35.
Q. E. D.

It will be very proper for young ſtudents to endeavour to obtain a variety of demonſtrations of one and the ſame propoſition, deduced from different principles; hereby they will be able to diſcover which are the moſt ſimple and elegant, and greatly improve their taſte and judgment. An inſtance of the great fecundity of Geometry in this reſpect is given in the beginning of a periodical work publiſhed a few years ago under the title of the BRITISH ORACLE, where there are given fifteen demonſtrations of one and the ſame Theorem, all independent of each other, and derived from very different principles and conſtructions. I ſhall now therefore proceed to give another demonſtration of the foregoing theorem, not as more ſimple than the preceding, but as leſs ſo, and further fetched. It was however given by an author of no ſmall note in the laſt century.

ANALYSIS.

With one of the angular points at the baſe as center, and the adjacent ſegment as radius, deſcribe a circle, which let cut the adjacent ſide in F, and the ſame produced in G, and the biſecting line in K. AB : BC :: AD : DC by VI. 3. and AB : BC :: AB^2 : AB×BC, and AD : DC :: AD^2 : ADC by VI. 1. Therefore by V. 19. AD : DC :: $AB^2 - AD^2$: AB×BC—ADC. Now $AB^2 - AD^2 = AB^2 - AF^2$ = GBF by II. 6. = DBK by III. 36. and AB×BC—ADC = BD^2 by hypotheſis (for AB×BC is put = BD^2 + ADC.) Therefore AD : DC :: DBK : BD^2. But DBK : BD^2 :: BK : BD by VI. 1. Therefore AD : DC :: BK : BD, which is true by Theorem LVIII. in the following collection.

SYNTHESIS.

AD : DC :: BK : BD (by the Theorem) :: DBK : BD^2 by VI. 1. Now DBK = GBF by III. 36. = $AB^2 - AF^2$ by II. 6. = $AB^2 - AD^2$. Therefore AD : DC :: $AB^2 - AD^2$: BD^2. Moreover AD : DC :: AD^2 : ADC by VI. 1. Therefore by V. 12. AD : DC :: AB^2 : BD^2 + ADC.

ADC. Now AD : DC :: AB : BC by VI. 3. and AB : BC :: AB^2 : AB×BC by VI. 1. Therefore by equality $AB^2 : BD^2 + ADC :: AB^2 : AB \times BC$, and by V. 9. $BD^2 + ADC = AB \times BC$.

Q. E. D.

Here it is evident that this demonſtration falls ſhort of the preceding, becauſe it does not flow immediately from an elementary propoſition, but in order to it's ratification the fore-cited Theorem LVIII. muſt be previouſly demonſtrated as a Lemma.

When a *Problem* is propoſed to be ſolved, we muſt apply our method of *Reſolution* thus. We muſt conceive the thing required to be already done, and from this ſuppoſition we muſt reaſon, deducing one conſequence from another, and proceeding ſtep by ſtep, till we can arrive at ſomething that is granted, ſomething that may be effected by means of the Poſtulates and Elementary Propoſitions, ſomething which (in the ſtyle of the Antients) is given, or a *Datum*. Which if we can do, we ſhall then be able to form

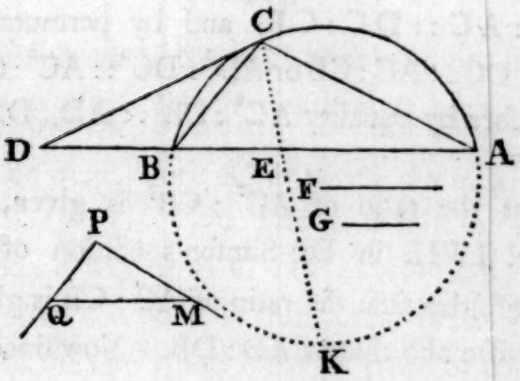
form our *Syntheſis* or *Compoſition* by making the *Datum*, we arrived at in the laſt ſtep of our Analyſis, the firſt ſtep or foundation of our Syntheſis; and then reaſoning in a retrograde order, and taking the ſame ſteps back again, we ſhall deduce one conſequence from another, till we arrive at the original *Quæſitum*, or thing required to be done in the Problem propoſed, which was the firſt thing laid down and ſuppoſed in our Analyſis.

Take the following example, being the 155th Propoſition of *Pappus's* VIIth book.

PROBLEM.

It is required in a given ſegment of a circle from the extremes of the baſe A and B to draw two lines AC and BC meeting at a point C in the circumference, and ſuch that they ſhall have a given ratio to each other, viz. that of F to G.

ANALYSIS.



ANALYSIS.

Suppoſe the thing done, and that the point C is found: then by way of preparation or conſtruction, or ſomething to found our Analyſis upon, let us ſuppoſe that a tangent to the ſegment at the point C is drawn, which meets AB produced in the point D. Now by hypotheſis AC : CB : : F : G, alſo $AC^2 : CB^2 : :$ AD : DB, which is thus proved.

Since DC touches the circle and BC cuts it, the angle DCB=BAC by III. 32. Alſo the angle D is common to both the triangles CDB and CDA, therefore they are ſimilar, and by VI. 4. AD : DC : : DC : DB, hence $AD^2 : DC^2$: : AD : DB by VI. 20. cor. But alſo by VI. 4.

AD

AD : AC :: DC : CB, and by permutation AD : DC :: AC : CB or $AD^2 : DC^2 :: AC^2 : CB^2$, therefore by equality $AC^2 : CB^2 :: AD : DB$.

But the ratio of $AC^2 : CB^2$ is given, (by Prop. LVII. in Dr. Simſon's edition of the *Data* *,) becauſe the ratio of AC : CB is given, therefore alſo that of AD : DB. Now ſince the ratio of AD : DB is given, therefore alſo, by *Data* VI. that of AD : AB, and hence by *Data* II. AD is given in magnitude.

And here the Analyſis properly ends. For it having been ſhewn that AD is given, or that a point D may be found in AB produced ſuch, that from it a tangent being drawn to the circumference, the point of contact will be the point ſought; we may now begin our Compoſition or Synthetical Demonſtration: which we muſt do by finding the point D, or laying down the line AD, which we affirmed to be given in the laſt ſtep of our Analyſis.

* *Dr. Simſon has altered the order of the Propoſitions of this book, but by marginal figures refered to the original order in the Greek text.*

SYNTHESIS.

SYNTHESIS.

Conſtruction. Make as $F^2 : G^2 :: AD : DB$ (which may be done, ſince AB is given, by making it as $F^2 - G^2 : G^2 :: AB : DB$, and then by compoſition it will be as $F^2 : G^2 :: AD : DB$) and then from the point D thus found draw a tangent to the circle, and from the point of contact C drawing CA and CB the thing is done.

Demonſtration. Since by *conſtruction* $F^2 : G^2 :: AD : DB$, and alſo $AD : DB :: AC^2 : BC^2$ (which has been already demonſtrated in the analyſis and may be here proved in the ſame manner.) Therefore $F^2 : G^2 :: AC^2 : BC^2$, and conſequently $F : G :: AC : BC$.

Q. E. D.

Here we ſee an inſtance of the method of *reſolution* and *compoſition*, as it was practiſed by the Antients, for the ſolution here given is that of *Pappus Alexandrinus*. But becauſe the method of referring and reducing every thing to the *Data*, and conſtantly quoting the ſame, may appear to many to be very tedious and troubleſome: and indeed it is unneceſſary to

thoſe who have already made themſelves maſters of the ſubſtance of this valuable book of *Euclid*, and have by practice and experience acquired a facility of reaſoning in ſuch matters; I ſhall therefore now ſhew how we may abate ſomething of the rigour and ſtrict form of the antient method of demonſtration, without diminiſhing any part of it's admirable perſpicuity and elegance. And this I ſhall do by inſtancing in another ſolution or two of the ſame problem.

But before I do this, it may be proper to take notice that, in this buſineſs of the reſolution of problems, every thing cannot be brought within ſtrict rules; nor any infallible directions given, whereby a man may be enabled to ſucceed in all poſſible caſes; but that there is need of a previous preparation, a kind of mental contrivance and conſtruction, in order to form a connexion between the *Data* and *Quæſita*, which muſt be left to every one's own ſagacity to find out. And it is on this very account chiefly that I recommend the exercise and employment of ſolving *Geometrical* Problems, as a means to help our invention, and to improve and

ſtrengthen

ſtrengthen our reaſoning powers, when we employ them on any other ſubject of a different nature. And here it is that the Geometrician ſhews his taſte and acuteneſs in chooſing the proper foundation, whereon to build the moſt elegant conſtruction and demonſtration; and the mathematical reader his judgment in being able, amidſt a variety, to diſtinguiſh the ſame. However ſome people may be apt to ridicule the notion of taſte, when applied to theſe ſubjects, yet I do maintain, that the word and thing ſignified thereby is very properly applicable thereto: and I believe all true mathematicians will bear me out, when I affirm that a man may ſhew as much taſte in being able to diſtinguiſh between the different degrees of elegance in mathematical demonſtrations, as a ſtudent in the *Belles Lettres*, or a profeſſed critic, can, by his right reliſh for the *ſublime* or *pathetic* in poetry or oratory.

But to return: before we begin our analyſis, I ſay there is moſt commonly ſome previous preparation neceſſary, in order to form a connexion between the *Data* and *Quæſita*, which cannot

fall within any rules; but is various, according to the various nature of the problems propoſed. Right lines muſt be drawn in particular directions; or of particular magnitudes: biſecting perhaps a given angle, or perpendicular to a given line; tangents from a given point to a given curve; circles muſt be deſcribed from a given center with a given radius; or touching given lines or other given circles; or ſuch-like other operations. Whoever is converſant with the works of *Archimedes*, *Apollonius* or *Pappus*, knows very well that they all found their analyſis upon ſuch-like previous operations. Now the great ſkill of the Analyſt conſiſts in diſcovering the moſt proper *effections* whereon to found his Analyſis: and young ſtudents ought to exerciſe their abilities and improve their ſagacity therein: for the ſame problem may frequently be conſtructed many different ways, and many different demonſtrations given of the ſame; which I ſhall now exemplify by giving two more different ſolutions of the aforeſaid problem.

ANALYSIS.

ANALYSIS.

Let us ſuppoſe then again that the thing is done, *i. e.* AC : CB : : F : G, (ſee the former figure) and let the baſe of the ſegment AB be cut in the ſame ratio in the point E. Then EC being drawn will biſect the angle ACB by VI. 3. conſequently if the circle be completed and CE produced to meet it in K, the remaining circumference will alſo be biſected in K. Therefore the point K, as well as E, being given, the point C muſt alſo be given.

SYNTHESIS.

Conſtruction. Let the given baſe of the ſegment AB be cut in the point E in the aſſigned ratio of F : G by VI. 10. and compleat the circle by III. 25. biſect the remaining circumference in K by III. 30. join KE and continue it to meet the given circumference in C, and drawing CA, CB, the thing is done.

Demonſtration. Since the arc KC = the arc KB, the angle ACK = the angle BCK by III. 27. Therefore AC : CB : : AE : EB by VI. 3.

but

but AE : EB : : F : G by *conſtruction*; therefore AC : CB : : F : G.

Q. E. D.

Again this problem may be ſolved in another manner, by conſidering it in a little different light; for it is the ſame thing required as, having the baſe of a triangle given, together with the vertical angle and the ratio of the legs, to find the triangle.

Let P = the given angle, or in other words the angle in the given ſegment. Set off upon the legs of the angle P, PM and PQ in the aſſigned ratio of F : G, and join MQ. Then upon the given baſe AB make a triangle ACB ſimilar to MPQ by VI. 18. and the thing is done; and the demonſtration follows immediately from VI. 4.

Having now given a ſpecimen of what kind of ſolutions I would have the young ſtudent endeavour after, I have nothing further to add, but to adviſe him, when he undertakes a propoſition, not to be diſcouraged by one or two fruit-

fruitleſs attempts, and thereupon throw it aſide, or ranſack books for the ſolution; but if he would profit by this exerciſe, let him perſevere, encouraged by the words of *Syrus* in the play;

Nil tam difficile eſt quin quærendo inveſtigari poteſt.

N. B. Of the propoſitions here annext for exerciſes, the firſt thirty-ſeven theorems have figures to explain the ſame, contained in two copper-plates: the reſt of the theorems and the problems were purpoſely ſelected ſuch as may be ſufficiently explained by words alone, ſo that the reader may eaſily ſupply the figure. This was done that the expence might not exceed the pockets of thoſe for whom theſe exerciſes are principally calculated. Several of the theorems may ſeem to be dependent upon ſome of the preceding ones, and may indeed be moſt commodiouſly deduced therefrom, which I ſhall here enumerate, viz. II. IV. V. VII. VIII. X. XII. XIII. XIV. XV. XVII. XX. XXII.

XXII. XXIV. XXVI. XXVII. XXVIII. and XXXV. But the demonſtrations of all the reſt are independent and may be derived immediately from the Elements; as may alſo the demonſtrations of ſome of thoſe enumerated, which it will be worth the ſtudent's while to try.

GEOME-

GEOMETRICAL THEOREMS.

PROP. I.



IF a right line AB be biſected in E, and two points C & D taken therein ſuch that AC : CB :: AD : DB; then I ſay the rectangle DCE = the rectangle ACB.

The converſe of this propoſition is alſo true, which is this.

If a right line AB be biſected in E, and two points C and D taken therein ſuch that DCE = ACB; then I ſay AC : CB :: AD : DB.

PROP. II.

If in A B the diameter of a circle two points C and D be aſſumed ſuch that AC:CB::AD:DB, and from D an indefinite perpendicular to the ſame diameter as L D be erected, and through C any line be drawn to cut the ſame in E, and the circle in F and G; I ſay FC:CG::FE:EG.

The converſe of this propoſition is alſo true, which is this.

If any right line as L D be drawn perpendicular to the diameter A B of any circle and meets the ſame in D, and if from a point in the ſame diameter, as C, any line be drawn to meet the ſame perpendicular in E, and the circle in F and G, ſo that FC : CG :: FE : EG; I ſay that AC : CB :: AD : DB.

PROP. III.

Let there be a triangle A B C, whoſe baſe B C is biſected in D, and through the vertex A a line A E drawn parallel to B C, and anv line drawn through D to meet A B, A C, A E in F, G, H; then I ſay GD : DF :: GH : HF.

PROP. IV.

PROP. IV.

If in AB the diameter of a circle two points C and D be taken ſuch that AC:CB::AD:DB, and through the point D any line be drawn to meet the circle in E and F, and CE, CF be joined; then I ſay EC : CF :: ED : DF.

PROP. V.

If the baſe BC of a triangle be biſected in D, and through the vertex A a parallel thereto be drawn, and from D a perpendicular to BC be drawn to meet the parallel in E, and through D any line be drawn to meet AB, AC in F and G, and EF, EG be joined; then I ſay EF : EG :: FD : DG.

PROP VI.

If in the line AB be taken two points C and D ſuch that AC : CB :: AD : DB, and AE, BF be drawn perpendicular to AB, and through the point C be drawn any line to meet AE, BF in G and H, and DG, DH be joined; then I ſay that DG : DH :: GC : CH.

PROP. VII.

If in the diameter of a circle AB be taken any point C, and CDE be drawn meeting the circle in D and E, and DF be perpendicular to AB meeting it F, and the circle again in G, and EG be joined meeting AB in H; I ſay that AC : CB :: AH : HB.

Alſo, as the converſe, that if in the diameter AB two points be taken as C and H ſuch that AC : CB :: AH : HB, and from the points C and H two lines CE, HE be inflected to any point of the circumference E meeting the ſame again in D and G; when DG is drawn, it will be perpendicular to AB.

PROP. VIII.

If in the diameter of a circle AB two points C and H be taken ſuch that AC : CB :: AH : HB, and from the points C and H be inflected to any point of the circumference E two lines CE, HE meeting the ſame again in D and G; I ſay that EC : CD :: EH : HG.

PROP. IX.

If in AB the diameter of a circle be taken any point C, and CD be drawn meeting the circumference in D and E, and from the point D be drawn DF perpendicular to CD, which meets the diameter AB in F and the circumference in G; then I ſay that DC: CE:: DF: FG.

PROP. X.

If in AB the diameter of a circle two points C and D be taken ſuch that AC : CB :: AD : DB, and through the centre E a perpendicular to AB be drawn, and from C a line be drawn to meet the ſame in F, and if through D any line DG be drawn to meet the circle in G and H, and from the point G be drawn GK the ſame ſide of DG as F is of the diameter AB to make the angle DGK equal to the angle CFE, and let the line GK meet the circle in L and the line CF in M; then I ſay that GM : ML :: GD : DH.

PROP. XI.

If from any point C in the diameter of a circle produced a perpendicular be raiſed and from

from any point D in the ſame a line be drawn to cut the circle in E and F; then I ſay the rectangle EDF is equal to the rectangle ACB together with the ſquare of CD.

PROP. XII.

If from any point C in the diameter of a circle produced a perpendicular be raiſed and thereon CD be taken whoſe ſquare is equal to the rectangle ACB, and CE be put equal CD, and from any point in DE as H a line be drawn to cut the circle in F and G; then I ſay twice the rectangle FHG is equal to the ſum of the ſquares of HD and HE.

PROP. XIII.

If in AB the diameter of a circle two points C and D be ſo taken that, C being without, and D either within or without the circle, the ſquare of CD be equal to the rectangle ACB, and from C a perpendicular to AB erected, and any line drawn through D to cut the ſame in G and the circle in E and F; then I ſay the ſquare of GD will be equal to the rectangle EGF.

The

The converſe is alſo true, which is this.

If GC be perpendicular to AB the diameter of a circle and meets it without the circle in C, and if from G a line be drawn to cut the circle in E and F, and the diameter either within or without in D, and the ſquare of GD be equal to the rectangle FGF; then I ſay the ſquare of CD will be equal to the rectangle ACB.

PROP. XIV.

Things remaining as in the laſt propoſition, if the perpendiculars Eg and FH be demitted; then I ſay that the rectangle gCH is equal to the ſquare of CD.

PROP. XV.

If from C any point in the diameter of a circle AB produced a tangent be drawn, and from the point of contact D a perpendicular to the diameter DE be demitted; then I ſay that AC : CB :: AE : EB.

Or

Or converſely thus:

If in AB the diameter of a circle be taken two points C and E ſuch that AC: CB:: AE: EB, and from E a perpendicular ED raiſed, and CD drawn; then I ſay CD touches the circle in D.

Or thus:

If in AB the diameter of a circle produced a point C be taken, and therefrom a tangent as CD be drawn, and in the diameter a point E be taken ſuch that AC : CB :: AE : EB; then I ſay ED being drawn will be perpendicular to the diameter AB.

PROP. XVI.

Let AB be any chord in a circle and CD another cutting the former in E, CB being joined, from D draw DF parallel to CB to meet AB in F; I ſay that the rectangle AEF is equal to the ſquare of DE.

PROP. XVII.

If ABC be a triangle inſcribed in a circle whoſe ſides CA and CB are equal, and the rectangle

rectangle CBD equal to the ſquare of AB, and let AE be any line cutting CB in F and the circle again in E, and from E let a parallel to AB be drawn to meet CB in G; then I ſay that the rectangle CFG : BF^2:: CG : BD.

PROP. XVIII.

Let ABC be a triangle inſcribed in a circle, whoſe ſides AB and AC are equal, and from A any line be drawn meeting the circle again in D and BC in E; I ſay that the rectangle DAE is equal to the ſquare of AB.

PROP. XIX.

Things remaining as in the laſt propoſition, if lines touching the circle in A and C be drawn to meet in F, and FD be drawn cutting BC in G; I ſay that the rectangle BCG is equal to the ſquare of CE.

PROP. XX.

Let ABC be a triangle inſcribed in a circle whoſe ſides AB and AC are equal, and let AD be parallel to BC, and taking any point therein

D; let the rectangle under AD and P be equal to the ſquare of AB or AC, and from the points A and D let the lines AE, DE be inflected to any point E in the circle, meeting BC in F and G; I ſay the rectangle under FG and P = the rectangle BFC.

PROP. XXI.

If in AB the diameter of a circle be taken two points C and D ſuch that AC:CB::AD:DB, and D be within the circle, and DE be perpendicular to AB meeting the circle in E and F, and if through C any line be drawn meeting the circle in G and H, and the line DE in K, and GL touch the circle in G, and meet DE in L; then I ſay the rectangle LDK is equal to the ſquare of DE.

PROP. XXII.

If in AB the diameter of a circle be taken two points C and D ſuch that AC:CB::AD:DB, and D be without the circle, and DE be perpendicular to AB, and through C be drawn any line meeting the circle in G and H, and

the

the line DE in K, and GL touch the circle in G, and meet DE in L; then I ſay the rectangle LDK is equal to the rectangle ADB.

PROP. XXIII.

If AB be the diameter of a circle and CD perpendicular thereto meeting it in C, and from the points A and B be inflected AE, BE to any point E in the circumference, meeting CD in F and G; I ſay the rectangle GCF is equal to the rectangle ACB.

PROP. XXIV.

In AB the diameter of a circle let two points C and D be taken ſuch that AC:CB::AD:DB, and the point D be within the circle, and DE be perpendicular to AB, meeting the circumference in E and F, and let through C any line be drawn meeting the ſame in G and H, and from the points G and H let GN, HN be inflected to any point in the ſame N, and let them meet DE in M and L; I ſay the rectangle LDM is equal to the ſquare of DE.

PROP. XXV.

Let AB be the diameter of a circle and CD perpendicular to the ſame meeting the circumference in C and D, and let E be the centre, and from C and D let CF, DF be inflected to any point F in the circumference meeting the diameter AB in G and H; I ſay the rectangle GEH is equal to the ſquare of the radius AE.

PROP. XXVI.

In AB the diameter of a circle let two points C and D be taken ſuch that AC:CB::AD:DB, and let D be without the circle, and DE perpendicular to BD, through the point C let any line be drawn meeting the circumference in F and G, and from the points F and G let FH and GH be inflected to any point H in the circumference meeting DE in K and L; I ſay the rectangle KDL is equal to the rectangle ADB.

PROP. XXVII.

In AB the diameter of a circle let be taken the point C, and CD be perpendicular to AB, meeting the circumference in D and N, in CD

let

let be taken two points E and F on the ſame ſide of C with D ſuch that the rectangle ECF may be equal to the ſquare of CD, and from the points E and F let EG, FG be inflected to any point G in the circumference, meeting the ſame in H and K, and let HK when drawn meet the diameter AB in L; then I ſay that AL : LB :: AC : CB.

PROP. XXVIII.

In AB the diameter of a circle produced let be taken the point C, and CD be perpendicular to AB, and therein be taken two points E and F on different ſides of C ſuch that the rectangle ECF, may be equal to the rectangle ACB, and from the points E and F let EG, FG be inflected to any point G in the circle, meeting the ſame in H and K, and let HK when drawn meet the diameter AB in L; then I ſay that AL : LB :: AC : CB.

PROP XXIX.

Let AB touch a circle in B, and any line AE be drawn equal to AB, and likewiſe from A

let

let any line be drawn to cut the circle in C and D, and let EC, ED be drawn meeting the circle again in F and G; then FG being drawn will be parallel to AE.

PROP. XXX.

Let AB touch a circle in B, and therein be taken two points E and F on the ſame ſide of A ſuch that the rectangle EAF may be equal to the ſquare of AB, and from A let any line be drawn meeting the circle in C and D, and EC, FD be drawn meeting the circle again in G and H; then GH being drawn will be parallel to AB.

PROP. XXXI.

Let AB touch a circle in B, and any line AE be drawn and therein be taken two points E and F on the ſame ſide of A ſuch that the rectangle EAF may be equal to the ſquare of AB, and from A any line be drawn to meet the circle in C and D, and EC, FD be drawn meeting the circle again in G and H; GH being drawn will be parallel to AE.

PROP. XXXII.

Through any point A within a circle let a line be drawn meeting it in B and E, and therein two points F and G be taken ſuch on different ſides of A that the rectangle FAG may be equal to the rectangle BAE, and thro' A any line be drawn meeting the circle in C and D, and FC, GD being drawn to meet the circle again in H and K; then HK being drawn will be parallel to AB.

PROP. XXXIII.

Let AB be a line without a circle, and from A and B two lines be drawn to touch the circle in C and D, and let the ſquare of AB be equal to the ſum of the ſquares of AC and BD, and from A any line be drawn to meet the circle in E and F, and BE, BF be drawn meeting the circle again in G and H; the points A, G, H, will be in a right line.

PROP. XXXIV.

Let AB meet a circle in C and D, and A be without and B within the ſame, and let the rectangle

rectangle CAD be equal to the ſquare of AB together with the rectangle CBD, and through A any line be drawn meeting the circle in E and F, and BE, BF be drawn meeting the circle again in G and H; then the points A, G, H, are in a right line.

PROP. XXXV.

From the extremes of AB let two lines AC, BD be drawn to touch a circle in C and D, and in AB let a point E be taken on the ſame ſide of A with B ſuch that the rectangle BAE may be equal to the ſquare of AC, and alſo in AB another point F on the ſame ſide of B with E ſuch that the rectangle EBF may be equal to the ſquare of BD, and through A any line be drawn meeting the circle in G and H, and BG, BH be drawn meeting the circle again in K and L; then the points L, K, F, are in a right line.

PROP. XXXVI.

If from A the vertex of a triangle ABC be drawn AD to any point D in the baſe, and DE be drawn parallel to AC, and DF to AB;

I ſay

I ſay the ſum of the rectangles BAF, CAF will be equal to the ſquare of AD together with the rectangle BDC.

PROP. XXXVII.

Let A and B be two points in the diameter of a circle whoſe centre is C, and let the rectangle ACB be equal to the ſquare of the ſemidiameter; biſect AB in D, and raiſe the perpendicular DM; from the point A draw AF to any point F in the circumference, and FE perpendicular to DM; then I ſay that the ſquare of AF is equal to twice the rectangle contained by AC and FE.

PROP. XXXVIII.

If any regular figure be circumſcribed about a circle, and from any point within the figure there be drawn perpendiculars to all the ſides of the figure; the ſum of the perpendiculars will be equal to the multiple of the ſemidiameter of the circle by the number of the ſides of the figure.

PROP. XXXIX.

Let there be any number of right lines interſecting in a point, and making all the angles about the point equal, and let any circle paſs through the ſame point; I ſay the circumference thereof will be divided by the interſecting lines into as many equal parts as there are lines.

PROP. XL.

If there be two triangles ABC, DEF, which have one angle A in one equal to one angle D in the other, and another angle B in the firſt equal to the ſum of the angles D and E in the ſecond; then ſhall the ſides AC, BC, DE, EF be proportional.

PROP. XLI.

The ſquare of the line biſecting the vertical angle of any triangle is a mean proportional between the differences of the ſquares of each ſide including that angle, and the ſquare of the adjacent ſegment of the baſe made thereby.

PROP. XLII.

If from the ſame point two tangents be drawn to a circle, and a line be drawn joining the points of contact, and another line to be intercepted between the tangents cut the foregoing which joins the point of contact, ſo as to be biſected in the point of interſection; then I ſay that the part of that line which is a chord of the circle will alſo be biſected by the ſame point.

And, converſely, if the chord cutting the line joining the points of contact be biſected by the point of interſection; then the continuation of the ſame to meet the tangents will alſo be biſected by the ſame point.

PROP. XLIII.

If from one of the equal angles of an iſoſceles triangle a perpendicular be drawn to the oppoſite ſide; then I ſay that the rectangle contained under that ſide and it's ſegment intercepted by the perpendicular and the baſe is equal to half the ſquare of the baſe.

PROP. XLIV.

If in a line AB two points C and D be taken; then I ſay that

$\overline{AB+AD} \times BC + BC^2 = 2\,ABC + BCD.$

And moreover that A C D B

$\overline{AB+AD} \times CD + CD^2 = 2\,ADC + BCD.$

PROP. XLV.

If from the vertical angle of any triangle two lines be drawn to make equal angles with the ſides containing it, and to cut the baſe; then I ſay that the ſquare of one ſide is to the ſquare of the other ſide as the rectangle under the ſegments of the baſe contiguous to the firſt ſide is to the rectangle under the ſegments contiguous to the other ſide.

PROP. XLVI.

If in AB the diameter of a ſemicircle any point C be taken, and from thence any line as CD drawn to meet the circumference in D, and a perpendicular DE be demitted; then I ſay that the ſquare of the line AC is equal to the

ſquare

ſquare of the line CD together with the rectangle under the ſum of the diſtances of C from A and C from B and the line AE, when C is taken in the diameter AB produced; but equal to the ſquare of CD together with the rectangle under the difference of the diſtances of C from A and C from B and the ſame line AE, when C is taken in the diameter itſelf.

PROP. XLVII.

If from one angle A of a rectangle ABCD a line be drawn to cut the two oppoſite ſides BC, DC, the former in F, and the latter produce in E; then I ſay that the rectangle EAF is equal to the ſum of the rectangles EDC, CBF.

PROP. XLVIII.

If a rectangle be inſcribed in a right-angled triangle, ſo that one of its angles coincide with the angle of the triangle; then I ſay that the rectangle under the ſegments of the hypothenuſe is equal to the ſum of the rectangles under the ſegments of the ſides about the right angle made by this inſcription.

PROP. XLIX.

PROP. XLIX.

If from the ſame point C two tangents be drawn to a ſemicircle whoſe diameter is AB, and if the extremes of the diameter and the points of contact be joined, either croſs-ways by two lines interſecting in F, or other-ways by two lines interſecting in H; then I ſay that CF or HC produced to meet the diameter AB will be perpendicular to the ſame.

PROP L.

If in a ſemicircle whoſe diameter is AB the chord of 60°. equal to the radius be inſcribed and from the center E a perpendicular drawn thereto and produced to meet the circumference in F; then I ſay that AF, EF, BF are continual proportionals.

PROP. LI.

If a line be cut in extreme and mean proportion; then I ſay that the ſquare of the whole, the rectangle under the whole and the greater ſegment, and the rectangle under the whole

and

and the leſſer ſegments are continual proportionals.

PROP. LII.

If three lines are continual proportionals; the ſum of the ſquares of the mean and the greater extreme is to the rectangle contained under the ſame, as the ſum of the extremes is to the mean.

PROP. LIII.

In every right-angled triangle, as the hypothenuſe is to the ſum of the ſides about the right angle, ſo is the ſaid ſum, to the ſum of the hypothenuſe and twice the perpendicular from the right angle.

PROP. LIV.

If a right line AD be any-ways cut in B, and from thence a perpendicular BE be erected equal to a mean proportional between the whole AD and the part AB, and a circle be drawn through the points A, D, E, and from A a per-

perpendicular be erected to meet the circumference in F; then I ſay that AF, AB, BE, AD are four continual proportionals.

PROP. LV.

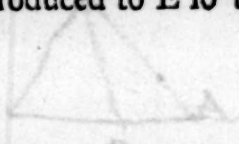
In every right-angled triangle, as the difference between the hypothenuſe and one ſide is to the difference between the ſame ſide and its adjacent ſegment, ſo is the ſame ſide to the ſame ſegment.

PROP. LVI.

If HC be a tangent to a circle meeting the diameter DB produced in H, and from the point of contact C a perpendicular CK to that diameter be drawn, and likewiſe a line from H cutting the circle in F and G, and the perpendicular CK in I, and F be the neareſt point to H; then I ſay that the ſquare of HF is to the ſquare of the tangent HC as FI to IG.

PROP. LVII.

If one ſide AC of an equilateral triangle ABC be produced to E ſo that CE may be equal to AC,



AC, and from A a perpendicular to AC raiſed, and from E a line drawn through the vertex B to meet the perpendicular in D; then I ſay that BD is equal to the radius of the circle which circumſcribes the triangle.

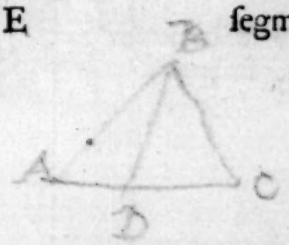
PROP. LVIII.

If BD biſect the vertical angle B of a triangle ABC and meet the baſe in D, and if with either of the other angular points A or C as center and the adjacent ſegment of the baſe as radius a circle be deſcribed to cut BD again in E; then I ſay that BE is to BD as that ſegment uſed as radius is to the other.

PROP. LIX.

If BD biſect the vertical angle B of the triangle ABC, and if on BA or BC from B be put a third proportional to the other ſide and the biſecting line; then I ſay the rectangle under that ſide on which it is put and its remainder when the third proportional is taken from it is equal to the ſquare of the adjacent

E ſegment



ſegment of the baſe made by the biſecting line. *i. e.* BCE=CD², or BAE=AD².

PROP. LX.

If an iſoſcles triangle be inſcribed in a ſemicircle and one of the equal ſides produced, and if from any point E in the diameter a perpendicular thereto be drawn to cut the ſide, the circle, and the ſide produced in the points G, H and F reſpectively; then I ſay that EG, EH and EF are continual proportionals.

GEOMETRICAL PROBLEMS.

PROB. I.

THROUGH a given point A without a given circle to draw a line to cut the circumference in two points X and Y ſuch, that the rectangle under the ſegments AX and XY may be equal to a given plane.

PROB. II.

Two circles being given neither touching, nor cutting, nor including one the other; to find a point in the line joining their centers ſuch that, tangents being drawn from thence to the circles, thoſe tangents may obtain a given ratio.

PROB. III.

Having a point C given in the diameter AB of a circle produced; it is required to find a

point X in the continuation BC ſuch that, a tangent being drawn therefrom, the ſaid tangent may be equal to XC.

PROB. IV.

A circle being given and two points either within or without it; through theſe two points it is required to draw another circle which ſhall biſect the periphery of the former.

PROB. V.

Having two points A and B given either both within or both without a given circle; it is required to inflect therefrom two lines AX and BX to the ſame point of the circumference X ſuch that, AX and BX meeting the circumference again in Y and Z, YZ may be parallel to AB.

PROB. VI.

Having a circle given, and likewiſe three points D, E, F, being in a right line, and either all three without the circle, or D and E within and F without; it is required from D

and

and E to inflect two lines DA and EA in ſuch a manner to the ſame point A of the ci cumference, to meet it again in B and C, that BCF may be a right line.

PROB. VII.

Four right lines being given, any three of which are greater than the remaining one; it is required of theſe four lines to conſtitute a quadrilateral figure which will ſtand in a circle.

PROB. VIII.

Having three points given in the circumference of a circle; to draw a diameter of that circle ſuch that, perpendiculars being let fall from the given points upon it, the ſegments of this diameter, intercepted by theſe perpendiculars, may bear a given ratio to each other.

PROB. IX.

Having two points given in the circumference of a circle, to draw a diameter of that circle ſuch that, perpendiculars being let fall from the given points upon it, the ſegment of this

this diameter, intercepted by theſe perpendiculars, may be equal to a given line.

PROB. X.

Having three points given in the circumference of a circle; to find a point from which three lines being drawn to the given points, theſe lines ſhall make angles with each other equal to two given angles.

PROB. XI.

Having a point given within a circle, and ſuppoſe therefrom an indefinite number of lines drawn to the circumference, and upon each of theſe lines as baſes iſoſceles triangles be conſtituted, one of whoſe equal ſides is a tangent to the circle; it is required to determine the locus of the vertices of thoſe iſoſceles triangles.

PROB. XII.

In a given ſegment of a circle to find a point in the circumference ſuch that, two lines being drawn from thence to the extremes of the baſe, the rectangle under theſe lines ſhall be equal to the ſquare of their difference.

PROB. XIII.

To produce the diameter of a given circle to ſuch a point that, a tangent being drawn from thence, and a perpendicular let fall from the point of contact upon the diameter, the rectangle under the tangent and ſuch perpendicular ſhall be to the rectangle under the diameter and the leſſer ſegment of the diameter made by the perpendicular in a given ratio.

PROB. XIV.

To find a point in the diameter of a given ſemicircle, or in the diameter produced, ſuch that, a line being drawn through it making a given angle with the diameter, and terminating in the circumference, the ſquare of that line ſhall be to the rectangle under the parts of the diameter intercepted by it in a given ratio.

PROB. XV.

Having a rhombus ABDC given; it is required from one of it's angles A to draw a line AG to meet one of the ſides produced in G, ſo that the whole line AG may be to the part of it EF intercepted between the diagonal CB and the ſide BD in a given ratio.

PROB. XVI.

Having a triangle ABC given whoſe ſides BA and BC are ſuppoſed indefinitely produced, and a point D given any where in the other ſide AC; it is required through D to draw a line IDH, intercepted between BA and BC, to cut off a triangle IBH equal to the original one ABC.

PROB. XVII.

Having a ſemicircle given, and a point D in it's diameter AC; it is required through D to draw another ſemicircle to be included in the former of ſuch a magnitude that, a tangent being drawn thereto from C as CEB, the part thereof EB, intercepted between the point of contact E and it's interſection with the former given ſemicircle B, may be equal to DA.

PROB. XVIII.

To divide a circle into any number of parts which ſhall be as well equal in area as in circumference.—N. B. *This may ſeem a paradox, however it may be effected in a manner ſtrictly geometrical.*

7 AP 53

FINIS.

Plate I.

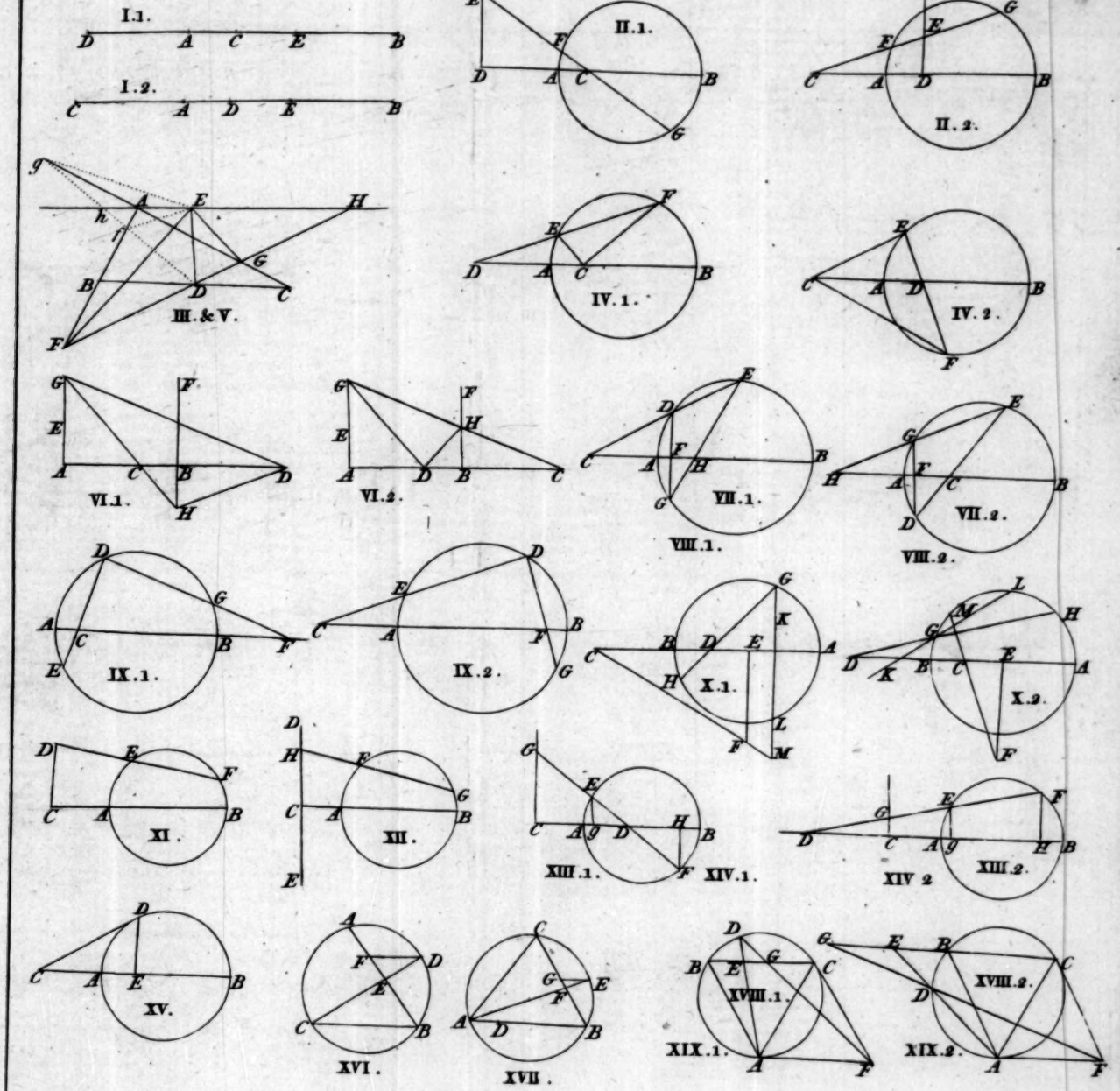
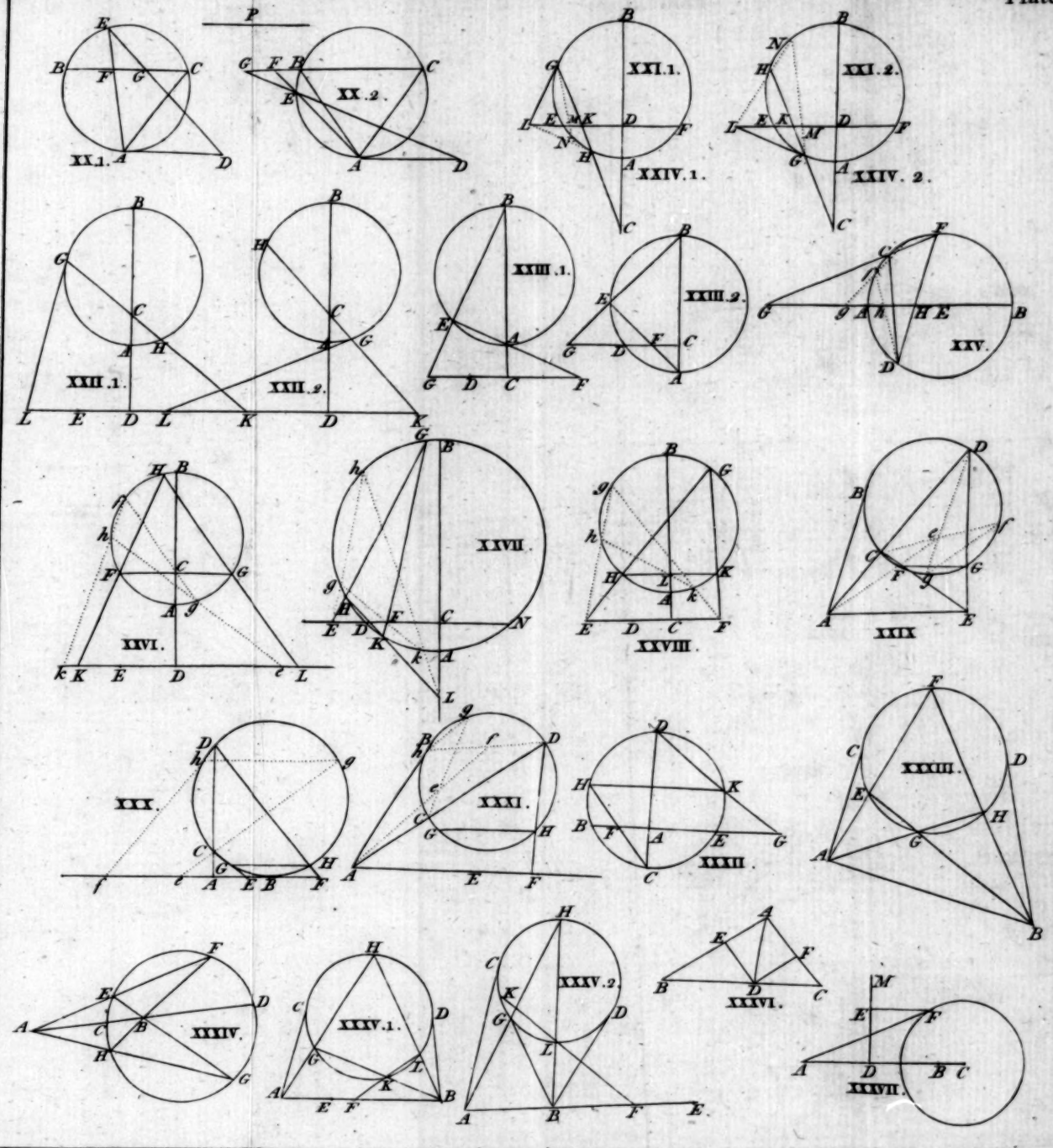

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I.2.
II.1.
II.2.
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IV.2.
VI.1.
VI.2.
VII.1.
VII.2.
VIII.1.
VIII.2.
IX.1.
IX.2.
X.1.
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XI.
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XIV 2
XV.
XVI.
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XIX.1.
XIX.2.

Plate II.


P
XX.1.
XX.2.
XXI.1.
XXI.2.
XXIV.1.
XXIV.2.
XXII.1.
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